# WAMC10

Ryan Tang, Wesley Wu, and more

October 19, 2024

## 1 Problems

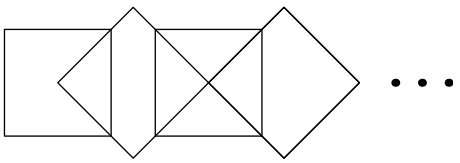
1. What is $\dfrac{1}{\frac{4}{\left(\frac{3}{4}\right)}} + \dfrac{\frac{\left(\frac{1}{4}\right)}{3}}{4}$?
   (A) $\frac{1}{24}$ (B) $\frac{1}{12}$ (C) $\frac{5}{24}$ (D) $\frac{17}{48}$ (E) $\frac{2}{3}$

2. Ryan has a number. He multiplies it by 5, which increases the number by 1. He then multiplies it by 3. What is the number now?
   (A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{15}{4}$ (E) $\frac{25}{4}$

3. Planes are supposed to descend to the runway at 141 knots. One flight descends to the runway at 241 knots. One knot is equal to 1.15 miles per hour. How much faster than normal, in miles per hour, is that flight descending?
   (A) 87 (B) 100 (C) 115 (D) 120 (E) 126

4. Spirit Airlines charges \$25 for a flight from Seattle to Las Vegas. However, Bob brought an extra carry-on item and is going to be charged an additional \$99. In addition, he wants to pick an exit row seat, which costs \$20. He also forgets to print out his tickets beforehand and must do so at the kiosk, costing \$2. Finally, on the flight, he buys some Pringles that cost \$4. What is the percent increase from his original ticket to his final total?
   (A) 300 (B) 400 (C) 420 (D) 500 (E) 600

5. 2024 congruent squares with side length 2 are drawn in a repeating manner. Below are the leftmost four of the 2024 squares in the figure. Compute the area of the union of all the squares.



   (A) 3036 (B) 6072 (C) 6073 (D) 8096 (E) 8097

6. If there exist positive integers $a, b \leq 5$ such that $6ab + 15a + 4b = 62$, find the value of $a + b$.
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

7. Wesley, Ryan, Alex, and Patrick are playing FTW for the first time in three months. In FTW, everyone must score a integer amount of points between 0 and 30, inclusive. Wesley scores 16 points in one game of FTW, while the second highest-scoring person has half that amount. How many scores in this game of FTW can there be? For example, one possible score is Wesley: 16, Ryan: 8, Alex: 5, Patrick: 0. Note that ties are allowed.
(A) 24 (B) 45 (C) 147 (D) 168 (E) 217

8. In a 4-digit number, the sum of the first two digits is 5, the sum of the middle two digits is 7, and the sum of the last two digits is 7. Find the sum of all possible values of this number.
(A) 10955 (B) 11190 (C) 14340 (D) 16260 (E) 16785

9. Suppose that $X$, $O$, $K$, $S$, $R$, and $B$ are distinct digits. Suppose

$$\overline{XOOKS} + \overline{RBO} = 10434.$$

Compute $X + O + O + K + S + R + B + O$.
(A) 8 (B) 14 (C) 17 (D) 18 (E) 21

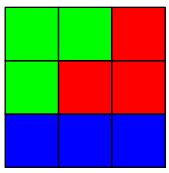
10. Positive integers $a, b, c, d$ satisfy $a+b+c+d = 20$ and $a^3+b^3+c^3+d^3 = 530$. Find the value of $a^2 + b^2 + c^2 + d^2$.
(A) 100 (B) 102 (C) 104 (D) 275 (E) 400

11. In the expansion of $(x^8+x^5+x^3+x^2+x+1)(x^8+x^5+x^3+x^2+x-2024)$, how many terms have a positive coefficient?
(A) 7 (B) 8 (C) 10 (D) 24 (E) 30

12. Call a positive integer an "emordnilap" if it reads the same when it is reflected about a horizontal line. For example, 18 is an emordnilap. How many five digit numbers are either palindromes or emordnilaps?
(A) 270 (B) 1620 (C) 1668 (D) 1860 (E) 2024

13. Find the sum of the digits of $116^2 - 85^2 + 12^2 + 9^2 + 4^2 + 3^2$.
(A) 7 (B) 10 (C) 15 (D) 16 (E) 19

14. How many ways are there to arrange the numbers 1 to 9 in this 3 by 3 grid so the numbers in the red squares sum to 7 and the numbers in the green squares sum to 22?



(A) 216 (B) 432 (C) 648 (D) 864 (E) 1296

15. In triangle $ABC$ with $\angle A = 90^\circ$, the altitude $AD$ is drawn with $D$ on $BC$. The angle bisectors of angles $BAD$ and $CAD$ intersect $BC$ at $P$ and $Q$, respectively. If $AB = 5$ and $AC = 12$, find $PQ$.
(A) 4 (B) $\frac{60}{13}$ (C) $\frac{63}{13}$ (D) $\frac{65}{13}$ (E) 5

16. Call a positive integer $n$ *flippers* if $n$, when written in base 3, contains an equal amount of 0's, 1's, and 2's. How many *flippers* numbers are there from 1 to $3^9$, inclusive?
(A) 1184 (B) 1440 (C) 1620 (D) 1776 (E) 1945

17. For how many ordered pairs of positive integers $(a, b)$ with $a, b \leq 100$ is the condition $|a - b| < \frac{|a+b|}{2}$ satisfied?
(A) 3300 (B) 3400 (C) 4500 (D) 6600 (E) 6700

18. Call a positive integer "orz" if it equals the sum of its digits cubed. For example, 5832 is orz because $5832 = (5 + 8 + 3 + 2)^3$. What is the sum of the first three orz integers?
(A) 1242 (B) 2710 (C) 5426 (D) 7622 (E) 11257

19. Ryan, Wesley, and Patrick are each uniformly randomly throwing one dart on a continuous number line from $-10$ to 10. Their score is defined as the distance from the number they hit to 3. What is the probability that the sum of their scores is less than 5?
(A) $\frac{4}{375}$ (B) $\frac{43}{3087}$ (C) $\frac{1}{48}$ (D) $\frac{11}{441}$ (E) $\frac{1}{24}$

20. Quadrilateral $ABCD$ has $AB = AD$, $CB = CD$, and $\angle B = \angle D = 90^\circ$. It is also given that $ABCD$ has an incenter and a circumcenter. The incenter of this quadrilateral is denoted $I$, and the circumcenter of this quadrilateral is denoted $O$. If $IO = 425$ and $AC = 1550$, compute the inradius of $ABCD$.
(A) 310 (B) 336 (C) 425 (D) 480 (E) 552

21. Let $a, b, c$ be the roots of $f(x) = x^3 - 4x^2 + 5x - 8$. Find $f(ab)f(bc)f(ac)$.
(A) $-71424$ (B) $-46080$ (C) $-21504$ (D) 10 (E) 81

22. Two circles with radius 19 and 7, respectively, are externally tangent to each other. A line $\ell$ passing through both circles intersects the larger circle at $A$ and $B$ and the smaller circle at $C$ and $D$ with $B$ on $AC$ and $C$ on $BD$. If $AB = BC = CD$, compute the length of $AD$.
(A) 11 (B) 18 (C) 24 (D) 27 (E) 33

23. In Yaptrick's triangle, it starts with a 1, and then the next term is one more than the sum of the two terms above it. The numbers on the edge

are always 1. The first few rows look like this:

$$\begin{array}{cc} 1 & \text{(Row 0)} \\ 1\ 1 & \text{(Row 1)} \\ 1\ 3\ 1 & \text{(Row 2)} \\ 1\ 5\ 5\ 1 & \text{(Row 3)} \end{array}$$

How many times does the number 239 appear in Yaptrick's triangle?
(A) 2 (B) 3 (C) 4 (D) 6 (E) 8

24. In isosceles trapezoid $ABCD$ with $AB$ and $CD$ parallel which has an incircle, the diagonal $BD$ intersects the incircle of $ABCD$ at points $P$ and $Q$, with $P$ closer to $D$ than $Q$ is. Given that $DP = 3$ and $QB = 1$, the length of $PQ$ can be expressed as $\sqrt{a + b\sqrt{c}} - d$, where $a$, $b$, $c$, and $d$ are positive integers, and $c$ is square-free. Compute $a + b + c + d$.
(A) 25 (B) 92 (C) 176 (D) 224 (E) 358

25. For how many ordered triples $(a, b, c)$ of positive integers $1 \leq a, b, c \leq 30$ is $\frac{(a+b+c)^2}{a^2+b^2+c^2}$ noninteger?
(A) 26913 (B) 26925 (C) 26946 (D) 26970 (E) 27000